Phase transitions in evolutionary games combinig two or three pair coordinations on a square lattice

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Multiagent evolutionary potential games with pair interactions when players are located on the sites of a lattice are equivalent to classical spin models if strategy updates are governed by the logit rule [1]. The concept of payoff matrix decomposition has revealed that such a suitable potential can only exist if the defining two-player game is composed of non-strategic self- and cross-dependent or elementary coordination-type game components [2]. Following our earlier systematic investigations we studied games whose pair interactions are composed of two or three equally strong non-overlapping elementary coordinations (i.e. Ising-type interactions) between strategy pairs. According to Monte Carlo simulations performed on a square lattice both of these models possess a continuous order-disorder phase transition that is accompanied by the spontaneous breaking of the symmetry of one of the coordinated strategy pairs. The ordered states of these systems can be characterized by two order parameters that vanish differently when approaching the critical point. In the four-strategy evolutionary game the system exhibits a critical phase transition where the first order parameter reproduces Ising type behavior when the temperature approaches the critical point while the second order parameter shows different algebraic behavior. As this model can be considered as a special case of the Ashkin-Teller model [3,4] with vanishing four-spin interactions, we could exactly determine the critical point with the utilization of a duality relation. The Monte Carlo simulations also indicated that the fluctuations of both order parameters diverge with the same critical exponent in the vicinity of the critical point. For the six-strategy version of these models a significantly different critical behavior was observed. Applying the method of mean-field approximation gives a qualitatively correct description of both the four- and six-strategy games and also predicts the presence of a first order phase transition for similar models with high enough strategy numbers.

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